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# Density of states for band random matrices with electric field

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**Abstract.** We present a long-range hopping tight-binding model in which the ensemble averaged density of states,  $\rho(E)$ , undergoes a gradual transition from a homogenous form with almost no variation to strongly peaked behaviour as an external electric field grows. The model consists of  $N \times N$  band random matrices of bandwidth  $b$ . The corresponding matrix elements,  $h_{ij}$ , have all vanishing average except on the diagonal where,  $\langle h_{ij} \rangle = \alpha i$ . Here, the parameter  $\alpha$  plays the role of the electric field. We approximate the behaviour of the width,  $\sigma_E$ , of the local density of states,  $\rho_L(E)$ , and use it to predict the value of  $\alpha$  around which the transition is centred.

## 1. Introduction

Random matrices serve as models for a wide variety of physical systems relevant to solid state physics [1], liquid theory [2], nuclear physics [3], atoms [4], molecules [5], and chaotic Hamiltonians [6]. In general, these apply to complex systems where details of interactions are either unavailable or unnecessary for a meaningful understanding of its properties. Instead, a statistical representation of the Hamiltonian is realized in the form of a random matrix, whereby the quantities of interest are obtained by an ensemble average over such representations [7]. For example, the energy level spacing statistics,  $P(s)$ , for a strongly chaotic, time-invariant, spinless Hamiltonian can be obtained from an ensemble of  $N \times N$  symmetric random matrices whose members contribute  $N - 1$  eigenvalue spacings each to the overall distribution. In the simplest model, known as the Gaussian orthogonal ensemble (GOE), the corresponding matrix elements are normally distributed with zero average and variance which is twice as large for the diagonal elements than for the off-diagonal ones. Eigenstates of the GOE have elements with fixed variance and therefore the overlap between any pair of such states is large. The corresponding energy levels repel each other and this leads to a  $P(s)$  that is very close to a Wigner distribution.

Around the same time as the appearance of the GOE, Wigner [8] introduced another ensemble whose matrix elements are normally distributed such that

$$\langle h_{ij} \rangle = \alpha i \delta_{ij} \tag{1a}$$

$$\sigma_{ij} \equiv \langle (h_{ij} - \langle h_{ij} \rangle)^2 \rangle = \begin{cases} (1 + \delta_{ij})\sigma^2 & |i - j| < b \\ 0 & \text{otherwise} \end{cases} \tag{1b}$$

where, for odd  $N$ , the indexes  $i, j$  run from  $-(N - 1)/2$  to  $(N - 1)/2$ . This is the banded random matrix ensemble (BRME). It has been used in the analysis of several different

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problems [9–13]. In particular, we have used the BRME as a further refinement of the GOE in modelling ergodic Hamiltonians [10–13]. Due to semi-classical constraints, the latter display a structure which, in turn, is mimicked by the BRME. For a particular Hamiltonian, the values of the parameters  $b$  and  $\alpha$  can be determined via relations of the Weyl type.

The ensemble of (1) can also be interpreted as a long-range hopping tight-binding model for an electron on a one-dimensional disordered lattice under the influence of a constant electric field,  $\alpha$ , that is

$$H_\alpha = \frac{p^2}{2m} + V(x) + \alpha x. \quad (2)$$

While tight-binding models (TBM) are easier to study than  $H_\alpha$  itself, they are known to behave somewhat differently from the latter. TBMs display discrete spectra and localized states [14]. On the other hand, it was rigorously proven that the eigenstates of  $H_\alpha$  are extended and its spectrum is continuous [15]. This discrepancy is due to the single-band approximation underlying the derivation of TBMs. It is assumed that the energy gap between consecutive bands is much larger than their width and that, consequently, no mixing between bands occurs. However, since in an infinitely long system the electric potential term in  $H_\alpha$  is unbounded, all the bands overlap and the TBM approximation clearly fails. This description is best illustrated in the *sloping-band* picture in which the consequence of the electric field is to make the energy bands linearly vary with the coordinate. In turn, however, this picture relies on the assumption of a characteristic length-scale separation between the atomic and the electric potentials [16].

Both the results of [15] and the failure of the TBM approximation do not necessarily carry over to the case of a system of finite size,  $N$ . In particular, if the total variation of the potential energy term,  $N\alpha$ , is much smaller than the zero field energy gap to adjacent bands,  $E_G$ , one expects the appropriate tight-binding model to accurately reproduce the properties of  $H_\alpha$ . Accordingly, we introduce the BRME as a tight-binding model for short, disordered 1D wires in strong electric fields such that  $N \ll E_G/\alpha$ . In contrast with previously studied TBMs, the BRME contains practically no assumptions aside from the single-band approximation. First, the hopping range is a free parameter rather than the standard assumption of only nearest-neighbour hopping. Second, the hopping matrix element is a random variable which for the case of a disordered system, is a more natural choice than the usual constant hopping. We therefore expect that phenomena occurring in the BRME will also occur in  $H_\alpha$  if the single-band approximation holds in the relevant range of parameters.

The purpose of this paper is to study the behaviour of the ensemble averaged density of states,  $\rho(E)$ , for the BRME. In particular, we find that a transition occurs in the  $\rho(E)$ , between a homogeneous form and a strongly peaked behaviour. While at small values of the field,  $\rho(E) = 1/N\alpha$  is constant, for large  $\alpha$  it is very small everywhere except for an  $O(\sigma)$  range around the Stark ladder levels,  $E_i = \alpha i$ . This transition corresponds to the breakdown of the *sloping-band* picture. In the next section, we describe the detailed properties of this transition and in section 3 possible extensions to this work are discussed.

## 2. Results

In the absence of electric field,  $\alpha = 0$ , it was recently shown [17] that  $\rho(E)$  has the form of a semicircle. That is, whenever  $1 \ll b \ll N$ ,

$$\rho(E) = \frac{2}{\pi r^2} (r^2 - E^2)^{1/2} \quad (3)$$

where  $r^2 = 8b\sigma^2$ . This result is similar to the corresponding one for the GOE [8]. There,  $\rho(E)$  also takes the form of a semicircle for  $N \gg 1$ , only that  $r^2 = 4N\sigma^2$ . Deviations from (3) due to finite  $b$  take the form of Lifshitz tails. Generalized versions of the BRME with variances that decay smoothly from the diagonal also display semicircular densities of states. Specifically, (3) still holds subject to the substitution,  $2b\sigma^2 \rightarrow I_i \equiv \sum_j \sigma_{ij}^2$ , as long as  $I_i$  is the same for all  $i$ . Moreover, these results are valid for arbitrary probability distributions of the matrix elements as long as averages vanish and variances are finite.

As far as the eigenstates are concerned [10, 18–20], it is known on general grounds that these are exponentially localized for the BRME at zero field. The corresponding localization length,  $L$ , satisfies  $L = cb^2$ . For  $N = O(L)$ , finite size corrections become significant [18], and  $L_N = NF_1(x \equiv b^2/N)$ . In the  $N \gg L$  regime, the overwhelming majority of adjacent levels correspond to states which have vanishingly small overlap with each other. Such levels do not repel and this leads to a Poisson spacing distribution.

Now consider the behaviour for small but finite  $\alpha$ . In previous work [10, 11], we have shown that the spacing distribution,  $P(s)$ , for the BRME displays a transition between a Poisson shape and a Wigner one. This occurs when the local densities of states corresponding to consecutive diagonal blocks of size  $L$  become centred at energies which, due to the electric field, become further apart from each other than their width. On the high field side of this transition, levels corresponding to states with small spatial overlap are energetically separated and therefore cannot be adjacent. As a consequence, adjacent levels strongly repel and the Wigner spacing distribution is obtained. The ratio of the two energy scales involved in this argument,  $\gamma = L\alpha/(4\sqrt{2b})$ , uniquely determines the intermediate forms of the  $P(s)$ . Since  $L = b^2 f(y \equiv \alpha b^{3/2})$ , we conclude that  $\gamma(y)$  and the transition is centred around  $\gamma \simeq 1$ . On the other hand, the average density of states of the BRME is constant in this range of  $\alpha$  and shows no indication of the transition in  $P(s)$ .

As the field is further increased, another transition is observed, which however modifies simultaneously both  $P(s)$  and  $\rho(E)$ . It occurs when the average level spacing,  $\alpha$ , exceeds the energy width of the local average density of states,  $\rho_L(\tilde{E})$ , corresponding to a single site. In this transition, the spacing distribution evolves from a Wigner form to a Gaussian of width  $\sqrt{2}\sigma/\alpha$  that is centred at  $s = 1$ . In what follows, however, we shall focus on the behaviour of the density of states alone.

Members of the BRME may be written as  $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_1$ , where  $\mathbf{H}_0$  is a diagonal matrix with non-random elements varying uniformly between  $-(N-1)\alpha/2$  and  $(N-1)\alpha/2$ . Thus, for  $N$  sufficiently large, the energy range of the spectrum is  $O(N\alpha)$  if  $\alpha > 0$  and is  $4\sqrt{2b}$  at  $\alpha = 0$ . Consequently,  $\rho(E)$  is discontinuous at  $\alpha = 0$ . Moreover, for infinitely large matrices, the ensemble is invariant under  $\mathbf{H} \rightarrow \mathbf{H} + \alpha\mathbf{I}$ , where  $\mathbf{I}$  is the unit matrix [21]. Therefore,  $\rho(E)$  is periodic with period  $\alpha$ . If all the elements of an eigenvector  $v_E$ ,  $\mathbf{H}v_E = Ev_E$ , are shifted by  $n$  sites, the new vector,  $v'$ , satisfies  $\mathbf{H}'v' = (E - n\alpha)v'$ , where  $\mathbf{H}'$  is another member of the BRME obtained from  $\mathbf{H}$  by shifting both indexes of all its elements by  $n$ . At  $\alpha = 0$ , all the vectors  $v'$  corresponding to different  $n$  are degenerate with energy  $E$ . One therefore expects that the local average density of states,  $\rho_L(\tilde{E} \equiv E - n_{\max}\alpha, \alpha)$ , where  $n_{\max}$  is the site on which the eigenvector corresponding to the eigenvalue  $E$  is maximal, is a continuous function of  $\alpha$ . In particular, for small enough values of  $\alpha$ ,  $\rho_L(\tilde{E})$  is still a semicircle (see figure 1). Notice that the local density of states is crucial for inspecting the validity of the *slowing-band* picture [16]. Specifically, it represents a section through the *slowing band* at a fixed value of the coordinate,  $i = 0$  [22].

The transition in the averaged density of states from homogeneous to peaked behaviour is closely related to the breakdown of the *slowing-band* picture. For small electric field, the matrix elements of  $\mathbf{H}_1$  are much larger than the spacing between the levels of  $\mathbf{H}_0$ ,

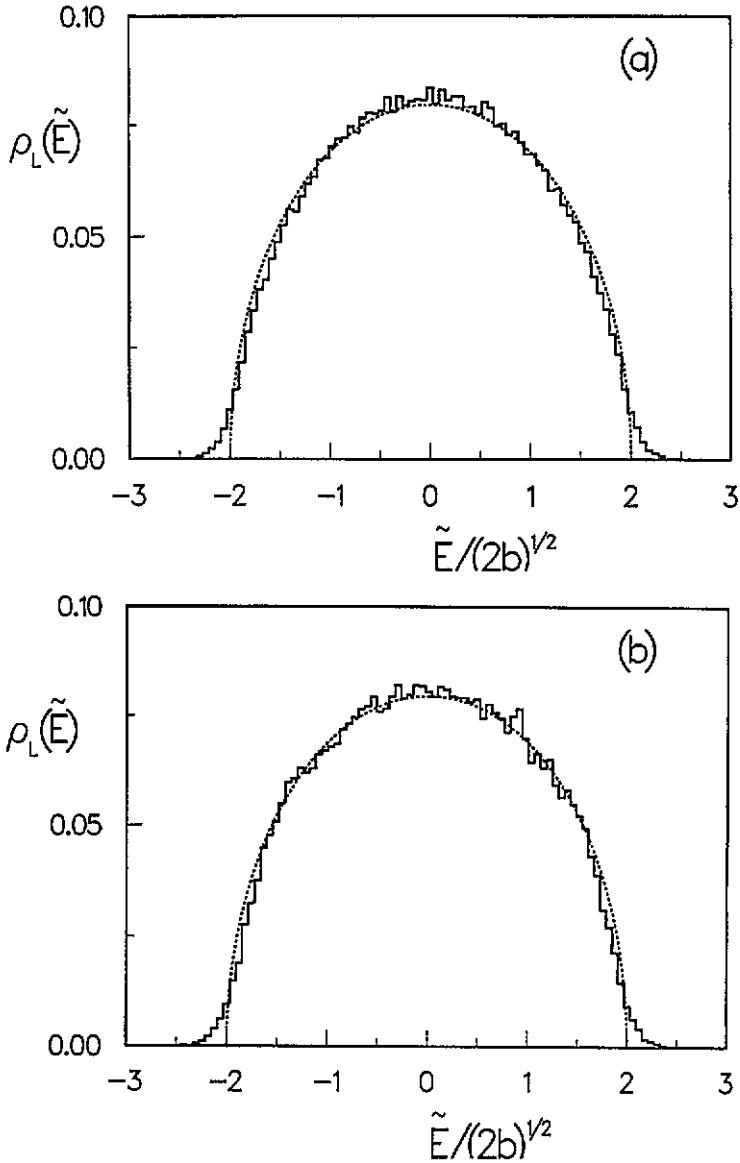


Figure 1.  $\rho_L(\tilde{E})$  as obtained numerically from an ensemble of 250 matrices with  $b = 8$ ,  $N = 400$  and (a)  $\alpha = 0$ , (b)  $\alpha = 0.1$  (histogram). Here and in all numerical calculations,  $\sigma = 1$ . The dashed curve is the prediction of (3). Clearly in (a),  $\rho_L(\tilde{E}) = \rho(E)$ .

$\sigma \gg \alpha$ , and therefore one expects that eigenvalues of  $\mathbf{H}$  are found at arbitrary  $E$  with equal probability. In this range the average density of states is constant,

$$\rho(E) = \frac{1}{N\alpha} \quad (4)$$

(see figure 2). On the other hand, in the limit  $\alpha \rightarrow \infty$ ,

$$\rho(E) = \frac{1}{N} \sum_{i=-(N-1)/2}^{(N-1)/2} \frac{1}{\sqrt{4\pi\sigma^2}} e^{-(E-i\alpha)^2/4\sigma^2} \quad (5)$$

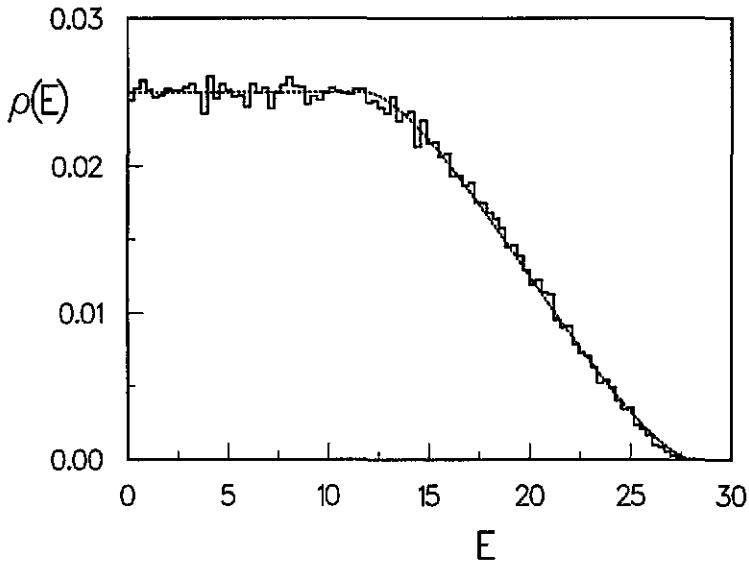


Figure 2.  $\rho(E)$  for the case of figure 1(b) (histogram). Since  $\rho(E) = \rho(-E)$ , only the range  $E > 0$  is shown. The broken curve is the prediction of (8).

which is negligible (but finite) everywhere except for a range of  $O(\sigma)$  around the Stark ladder energies,  $E_i = \alpha i$ . In both (4) and (5) we assume that  $N$  is very large and neglect the end effects which are bound to appear when  $E = O(N\alpha/2)$ .

In addition to being continuous at  $\alpha = 0$ , the local density of states,  $\rho_L(\tilde{E})$ , is also a useful instrument in the quantitative study of the transition between the homogeneous and peaked regimes. From the behaviour of eigenvectors under translations it follows that the probability of  $n_{\max}$  to take on any integer value between 1 and  $N$  is equal to  $1/N$ , barring finite size effects. Accordingly,

$$\rho(E) = \frac{1}{N} \sum_{n_{\max}=1}^N \rho_L(E - n_{\max}\alpha) \tag{6}$$

and thus one expects the transition in  $\rho(E)$  to be centred around the value of  $\alpha$ ,  $\alpha_C$ , where the width of  $\rho_L(\tilde{E})$ ,  $\sigma_{\tilde{E}}$ , becomes equal to  $\alpha$  itself,

$$\sigma_{\tilde{E}}(\alpha_C) = \alpha_C. \tag{7}$$

In order to find  $\alpha_C$ , we shall analyse the behaviour of  $\rho_L(\tilde{E})$  (and in particular its width) and its relation to  $\rho(E)$  for the various ranges of  $\alpha$ .

At small  $\alpha$ , the sum in (6) is well approximated by the corresponding integral. While for infinite matrices this approximation is equivalent to that of (4), the integral version of (6) renders the finite size behaviour of  $\rho(E)$  with high accuracy. Using (3) for  $\rho_L(\tilde{E})$ , one obtains to leading order in  $N$

$$\rho(E) = \begin{cases} \frac{1}{N\alpha} f_- & -N\alpha/2 - r < E \leq -N\alpha/2 + r \\ \frac{1}{N\alpha} & -N\alpha/2 + r < E \leq N\alpha/2 - r \\ \frac{1}{N\alpha} f_+ & N\alpha/2 - r < E \leq N\alpha/2 + r \end{cases} \tag{8a}$$

where

$$f_{\pm} = \frac{1}{2} \pm \frac{(E \mp N\alpha/2)}{\pi r^2} \sqrt{r^2 - (E \mp N\alpha/2)^2} \pm \frac{1}{\pi} \sin^{-1} \left( \frac{E \mp N\alpha/2}{r} \right) \tag{8b}$$

and  $r$  is the radius of the semicircle,  $\sqrt{8b}$ , and we have assumed that  $N\alpha/2 > r$  (see figure 2).

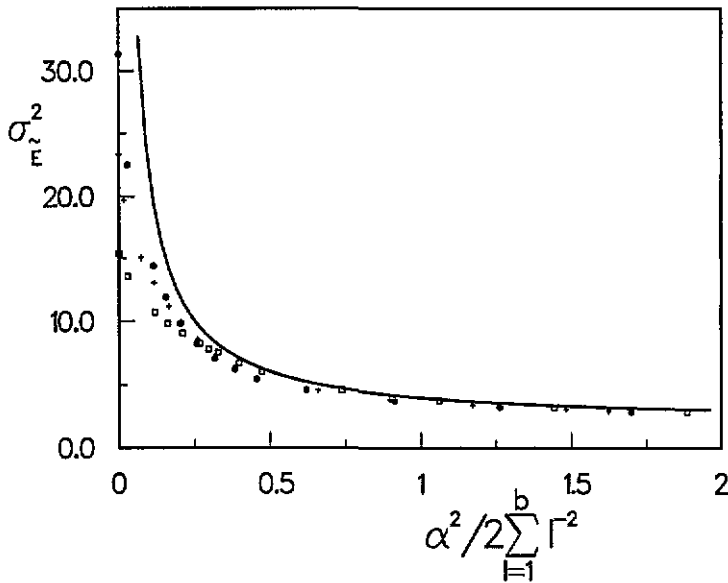
For  $\alpha \rightarrow \infty$ , (5) and (6) are clearly equivalent. Here the eigenvalues are just the diagonal elements and the corresponding eigenvectors have only one non-vanishing element,  $v_j = \delta_{ij}$ . Accordingly,  $\rho_L(\vec{E})$  is a Gaussian of width  $\sigma_{\vec{E}} = \sqrt{2}\sigma$  that is centred at  $\vec{E} = 0$ . We can obtain corrections to  $\rho_L(\vec{E})$  by considering the perturbation due to the off-diagonal matrix elements. To second order in perturbation theory, the eigenvalues may be expressed

$$E_i = \alpha i + (\mathbf{H}_1)_{ii} + \sum_{j=i-b}^{i+b} \frac{H_{ij}^2}{\alpha(i-j)}. \tag{9}$$

The average of  $E_i$  is simply  $\alpha(i - (N + 1)/2)$  (to any order). Accordingly,

$$\sigma_{\vec{E}}^2 = 2\sigma^2 + \sum_{j=i-b}^{i+b} \text{Var} \left[ \frac{H_{ij}^2}{\alpha(i-j)} \right] = 2\sigma^2 \left( 1 + \frac{2}{\alpha^2} \sum_{l=1}^b \frac{1}{l^2} \right). \tag{10}$$

Notice that, for  $b = \infty$ , the sum in (10) becomes  $\pi^2/6$ . In figure 3, we check the range of  $\alpha$  for which perturbation theory agrees with the numerical results. As one should expect, this range is smaller the larger  $b$  is. Although the algebra soon becomes tedious, expressions



**Figure 3.** The validity of (10) (full curve) is checked against numerical data. The variance of  $\rho_L(\vec{E})$ ,  $\sigma_{\vec{E}}$ , as a function of  $\alpha^2 / \sum_{l=1}^b 1/l^2$  for  $b = 8$  (square), 12 (cross), and 16 (bullet), and  $N = 90, 200,$  and  $356$ , respectively, such that  $x \equiv b^2/N = 0.72$ . The ensemble size is chosen to include  $10^3$  eigenvalues.

analogous to that of (10) can also be found for higher moments of  $\rho_L(\tilde{E})$ ,  $M_{2n}$ . These take the form [23]

$$M_{2n} = (2n - 1)!!(\sqrt{2}\sigma)^{2n} + \frac{C_{2n}}{\alpha^{2n}} \sum_{j=i-b}^{i+b} \frac{1}{(i-j)^{2n}} \quad (11)$$

where  $C_{2n}$  does not depend on either  $\alpha$  or  $b$ . According to (10) and (11), as  $\alpha$  gets smaller, the ratios  $R_{2n} \equiv M_{2n}/\sigma_{\tilde{E}}^{2n}$  grow and thus  $\rho_L(\tilde{E})$  becomes thinner and acquires larger tails in comparison to a Gaussian. Since for the semicircle the  $R_{2n}$  ratios are smaller rather than larger than those of the Gaussian,  $R_{2n}(\alpha = 0) = (2n - 1)!!(\sqrt{2}\sigma)^{2n}/(n + 1)!$ , this trend cannot continue indefinitely. In fact, at around  $\alpha^2 b \approx 11$ ,  $\rho_L(\tilde{E})$  momentarily recovers its Gaussian shape.

One can obtain an estimate for  $\alpha_C$ , the solution of (7), using the perturbative form of  $\sigma_{\tilde{E}}$ , (10). For  $b = \infty$ , the result,  $\alpha_C = 1.937 \dots$ , lies close to the edge of the range of validity of (10) (see figure 3). Therefore, in order to get a better estimate for  $\alpha_C$ , we need to obtain the behaviour of  $\sigma_{\tilde{E}}$  at arbitrary values of  $\alpha$ . The form

$$\sigma_{\tilde{E}}^2 = \frac{2b}{1 + c'\alpha^2 b} + \frac{2\alpha^2}{c + \alpha^2} \quad (12a)$$

where

$$c = \frac{1}{c'} - 2 \sum_{l=1}^b \frac{1}{l^2} \quad (12b)$$

satisfies both the small and large  $\alpha$  limiting behaviours and in addition, fits the numerical results reasonably well (see figure 4). On the other hand, finite  $b$  corrections tend to complicate the analysis of the numerical data. Accordingly, in figure 4, we replaced the

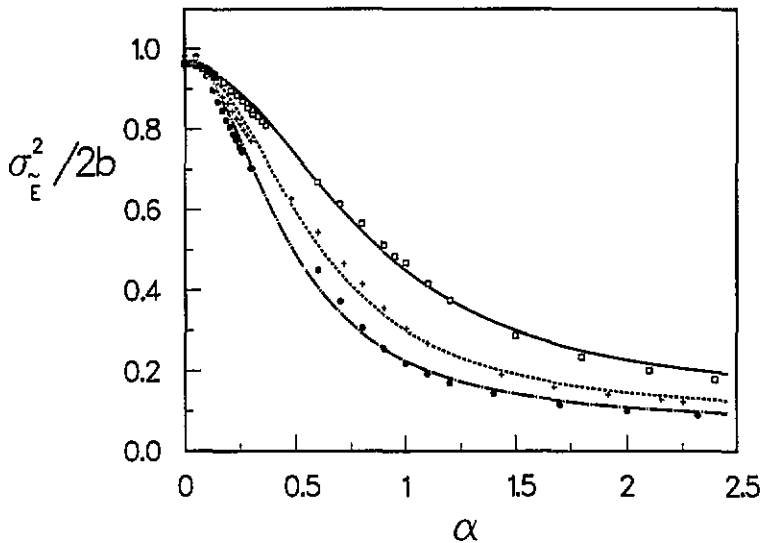


Figure 4. Same as in figure 3, only here (12) is fitted to the numerically obtained values of  $\sigma_{\tilde{E}}$  for  $b = 8$  (full curve),  $b = 12$  (broken curve), and  $b = 16$  (chain curve).



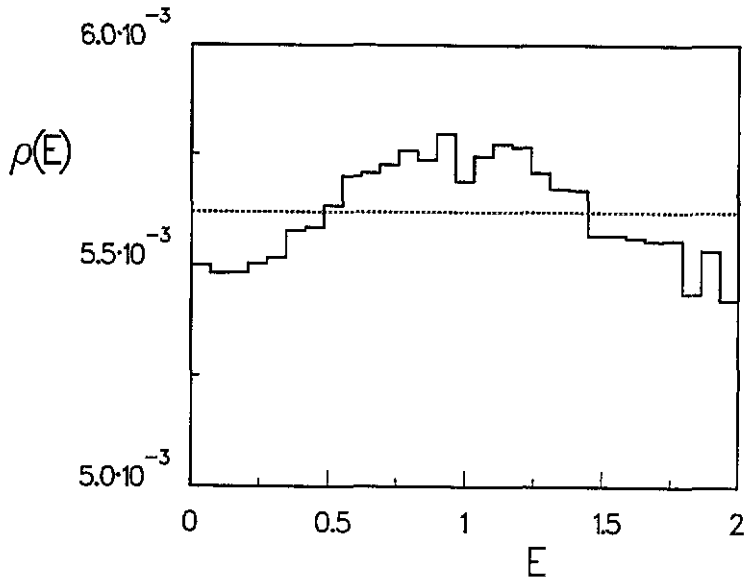


Figure 5.  $\rho(E)$  for  $E \in (0, \alpha)$  (histogram). Taking advantage of the periodicity of  $\rho(E)$ , the average is performed over both the ensemble and all the  $(i\alpha, (i+1)\alpha)$  intervals in the range  $(-N\alpha/2 + \sqrt{8b}, N\alpha/2 - \sqrt{8b})$ . The ensemble is such that the histogram contains  $8 \times 10^5$  eigenvalues. Here,  $b = 8$ ,  $N = 89$  and  $\alpha = 2$ . The broken curve represents the value of  $\rho(E)$  as given by (4).

$b = \infty$  form of  $\sigma_E^2(\alpha = 0)$  in the numerator of the first term of (12a),  $2b$ , with its exact value,  $2b - b^2/N + b/N$ . This in turn modifies (12b),

$$c = \frac{1}{c'} - 2 \sum_{l=1}^b \frac{1}{l^2} - \frac{b-1}{2c'N}. \quad (12c)$$

Moreover, we have kept the finite size variable fixed,  $x \equiv b^2/N = 0.72$ . Then a best fit is obtained when  $c' = 0.163$ ,  $0.226$  and  $0.289$  for  $b = 8$ ,  $12$  and  $16$ , respectively. Notice that the corresponding values of  $c$  are  $2.834$ ,  $1.178$  and  $0.218$  and therefore, one expects that  $c \rightarrow 0$  as  $b \rightarrow \infty$ .

From figure (4), one obtains  $\alpha_C = 1.940$ ,  $1.912$  and  $1.912$  for  $b = 8, 12$  and  $16$ , respectively, which seems to converge to a value of about  $1.91$ . Notice that this result is less than  $2\%$  away from the one previously obtained by perturbation theory. Moreover, for  $b \gg 1$ ,  $\alpha_C$  does not depend on  $b$ . This conclusion also emerges when using (12a) with  $c = 0$  to solve (7). For large  $b$ , this leads to  $\alpha_C = 2.97$ .

We can now proceed to the discussion of the incipient stages of the transition in  $\rho(E)$ . This takes place via the gradual amplification of an oscillatory correction to the  $\rho(E)$  of (4), starting at  $\alpha = 0$  (see figure 5). For small values of  $\alpha$ , however, this oscillation is masked by statistical noise. In figure 6, we plot the average absolute value of the difference between the numerically obtained  $\rho(E)$  and that of (4),  $D(\alpha)$ ,

$$D(\alpha) = N\alpha \left\langle \left| \int_0^\alpha dE \left| \rho(E) - \frac{1}{N\alpha} \right| \right| \right\rangle \quad (13)$$

where  $\langle \dots \rangle$  denotes averaging over both the ensemble and different  $(i\alpha, (i+1)\alpha)$  intervals. For values of  $\alpha$  close to  $\alpha_C$  but such that  $D(\alpha)$  exceeds the noise level we find that  $D(\alpha) = a_1 \alpha^{13/4}$  where  $a_1 \approx 2 \times 10^{-4}$ .

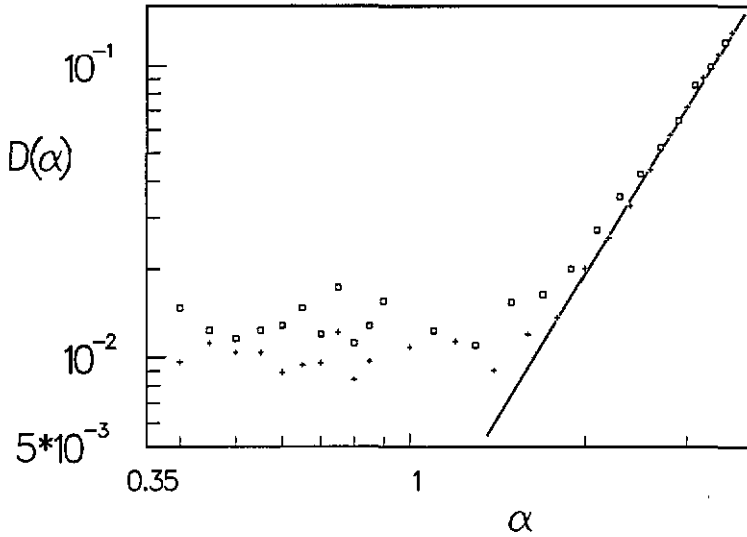


Figure 6.  $D(\alpha)$  for  $b = 8$  and  $N = 89$  (see (13)). Two ensemble sizes are used such that the number of eigenvalues in the first is  $10^5$  (square) and  $2 \times 10^5$  in the second (cross). The full line is,  $D(\alpha) = 2 \times 10^{-4} \alpha^{13/4}$ .

### 3. Conclusions

Although the numerical study of the BRME could be pursued even further, it appears that the results we have presented should now be used as the basis for an analytical approach. On the other hand, the two features which are most likely relevant to possible experiments are that: (a) the electric field leads to the thinning of the effective *sloping bands* (see figures 3 and 4) and (b) in the long range hopping regime,  $b \gg 1$ , the value of the electric field around which the transition is centred,  $\alpha_C$ , does not depend on  $b$ . Such experiments should consist of the spectroscopic investigation of *ensembles* of short, disordered wires subject to an external electric field. Although strictly speaking our model only applies to purely 1D systems, it is natural to expect that quasi-1D systems would behave in a similar manner. For example, one can renormalize the physical problem such that each atomic layer in the section of a wire corresponds to an effective single site in a 1D TBM. On the other hand, in most metals the condition for the validity of the one-band TBM limits the value of  $N$  needed to observe the transition in the average density of states,  $N_T$ , to be of  $O(1)$ . This problem can be avoided by using disordered (with random layer thickness) superlattice wires. Here, the ratio between the energy gap and the conduction band width,  $R_E \equiv E_G/E_B$ , can be made arbitrarily large by increasing the average distance between adjacent quantum wells. Since,  $N_T = O(R_E)$ , it should be possible to observe the homogeneous to peaked transition in the spectrum of such systems.

The most natural extension of this work is the study of the level spacing distribution,  $P(s)$ , in the range of parameters where the transition in  $\rho(E)$  occurs. While its general behaviour can be guessed from that of the  $\rho(E)$  itself, it has the advantage that it can be observed in an *individual* sample. That is, the ensemble average and the average over an energy range are equivalent for this quantity. This property, technically known in random matrix theory as *ergodicity*, is also shared by  $\rho_L(\tilde{E})$  but not by  $\rho(E)$  itself. Since the former is not directly measurable, the  $P(s)$  appears to be the optimal quantity for experiment and

it shall be further studied in future work [10, 11].

Another interesting possibility is to generalize the BRME to a multiple-band TBM. In such a model, one can study the transition between the regime with localized eigenstates and that with extended ones which occurs as bands pass through avoided crossings.

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- [21] Similarly, the BRME is invariant under  $H \rightarrow -H$  and as a consequence,  $\rho(E) = \rho(-E)$ .
- [22]  $\rho_L(\vec{E})$  is equivalent to the ensemble average of the standard local density of states,  $\rho_i(E) = \langle i | \delta(E - H) | i \rangle$ , for  $i = 0$ ,  $\rho_L(\vec{E}) = \langle \rho_0(E) \rangle$ . Notice that,  $\langle \rho_{i+k}(E) \rangle = \langle \rho_i(E - \alpha k) \rangle$ . While  $\langle \rho_i \rangle$  is more mathematically tractable, we feel that  $\rho_L$  is more intuitively appealing.
- [23]  $\rho_L(\vec{E})$  is even and thus all its odd moments vanish.